



PHYS 102 – General Physics II Midterm Exam 2

Duration: 90 minutes

Saturday, 12 April 2025; 09:30

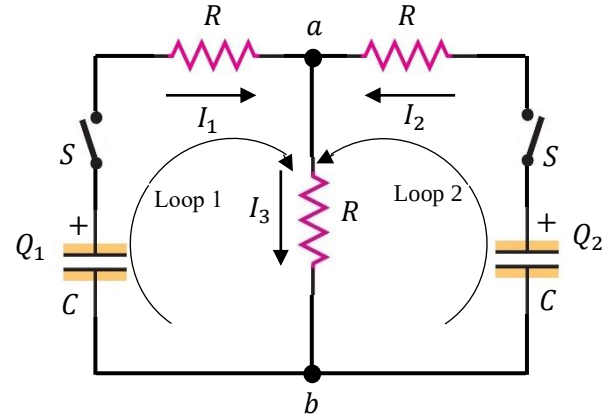
1. Consider a circuit containing two identical capacitors, each with a capacitance of C , and three identical resistors, each with a resistance of R , connected as shown in the figure. Initially, the charge on the left capacitor is Q_1 , and the charge on the right capacitor is Q_2 . Both switches are closed simultaneously at time $t = 0$.

(a) (5 Pts.) Write the junction condition for junction a in terms of the currents indicated in the figure.

(b) (5 Pts.) Write the loop equations for loop1 and loop 2 (Note the directions.)

(c) (5 Pts.) Find the time constant τ .

(d) (10 Pts.) Find the expression for the current I_3 for $t > 0$.



Solution:

(a) $I_1 + I_2 = I_3$

(b)

$$V_{C_1} - RI_1 - RI_3 = 0 \rightarrow \frac{q_1(t)}{C} - RI_1 - RI_3 = 0$$

$$V_{C_2} - RI_2 - RI_3 = 0 \rightarrow \frac{q_2(t)}{C} - RI_2 - RI_3 = 0$$

(c)

$$\frac{q_1(t) + q_2(t)}{C} - R(I_1 + I_2) - 2RI_3 = 0 \rightarrow \frac{q_1(t) + q_2(t)}{C} - 3RI_3 = 0$$

$$\frac{dq_1(t)}{dt} = -I_1, \quad \frac{dq_2(t)}{dt} = -I_2 \rightarrow \frac{dq(t)}{dt} = -(I_1 + I_2) = -I_3$$

where $q(t) = q_1(t) + q_2(t)$. So, we have

$$3R \frac{dq(t)}{dt} + \frac{q(t)}{C} = 0 \rightarrow q(t) = (Q_1 + Q_2)e^{-t/3RC}, \quad \tau = 3RC$$

(d)

$$I_3 = -\frac{dq(t)}{dt} \rightarrow I_3(t) = \frac{Q_1 + Q_2}{3RC} e^{-t/3RC}$$

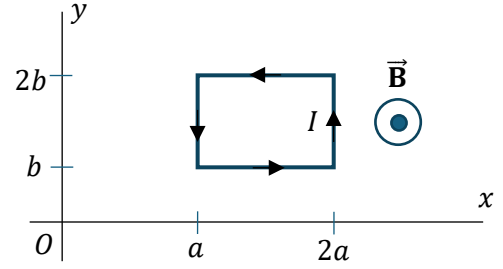
2. A rectangular current loop with sides a and b carries a current I in a **nonuniform** magnetic field. The loop is in the xy plane, as shown in the figure, perpendicular to a magnetic field which is out of the plane of the loop. At points in the plane, the field is given by $\vec{B} = (B_0 a/x)\hat{k}$, where B_0 is a constant.

(a) (5 Pts.) Determine the magnetic force on the left section of the current loop where $x = a$, and $b < y < 2b$.

(b) (5 Pts.) Determine the magnetic force on the bottom section of the current loop where $a < x < 2a$, and $y = b$.

(c) (5 Pts.) Determine the total magnetic force on the loop.

(d) (10 Pts.) If the magnetic field is given as $\vec{B} = (B_0 a/x)\hat{i}$, what is the magnitude of the magnetic torque on the current loop with respect to the center of the loop?



Solution:

$$d\vec{F} = I d\vec{\ell} \times \vec{B}$$

(a) On the left section where $x = a$, and $b < y < 2b$, magnetic field is constant.

$$\vec{B}(a) = B_0 \hat{k}, \quad d\vec{\ell} = -dy \hat{j}, \quad |\vec{F}_L| = IB_0 \int_b^{2b} dy = IB_0 b, \quad \vec{F}_L = -IB_0 b \hat{i}$$

(b) On the bottom section where $y = b$, and $a < x < 2a$,

$$d\vec{\ell} = dx \hat{i}, \quad d\vec{F}_B = I(dx \hat{i}) \times \left(\frac{B_0 a}{x} \hat{k}\right) = I \frac{B_0 a}{x} dx (-\hat{j}) \rightarrow |\vec{F}_B| = (IB_0 a) \int_a^{2a} \frac{dx}{x} = IB_0 a \ln 2$$

$$\vec{F}_B = -IB_0 a \ln(2) \hat{j}$$

(b) The magnetic force on the top and bottom sections of the loop cancel each other. On the right section

$$d\vec{\ell} = dy \hat{j}, \quad d\vec{F}_R = I(dy \hat{j}) \times \left(\frac{B_0}{2} \hat{k}\right) \rightarrow |\vec{F}_R| = \left(\frac{IB_0}{2}\right) \int_b^{2b} dy = \frac{IB_0 b}{2} \rightarrow \vec{F}_R = \frac{1}{2} IB_0 b \hat{i}$$

Therefore, the net force is

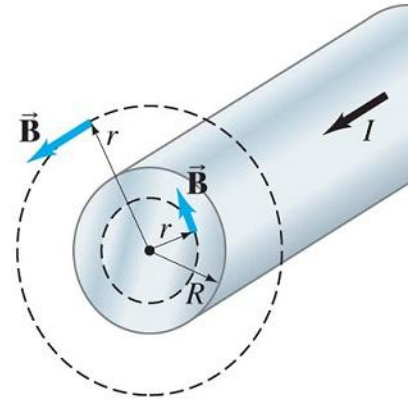
$$\vec{F} = -\frac{1}{2} IB_0 b \hat{i}$$

(c) If $\vec{B} = (B_0 a/x)\hat{i}$, magnetic force on the top and bottom sections are zero. For the left and right sections we have

$$\vec{B}(a) = B_0 \hat{i}, \quad d\vec{\ell} = -dy \hat{j}, \quad |\vec{F}_L| = IB_0 \int_b^{2b} dy = IB_0 b, \quad \vec{F}_L = IB_0 b \hat{k}, \quad \vec{F}_R = -\frac{1}{2} IB_0 b \hat{k}$$

$$|\vec{F}| = \left(\frac{a}{2}\right) (IB_0 b) + \left(\frac{a}{2}\right) \left(\frac{1}{2} IB_0 b\right) = \frac{3}{4} IB_0 ba$$

3. A very long straight cylindrical wire conductor of radius R carries a current I of uniform current density in the conductor. Determine the magnitude of the magnetic field due to this current at (a) (13 Pts.) points inside the conductor ($r < R$), and (b) (12 Pts.) points outside the conductor ($R < r$). Assume that r , the radial distance from the axis, is much less than the length of the wire.



Solution: Because the wire is long, straight, and cylindrical, we expect from symmetry that the magnetic field must be the same at all points that are the same distance from the center of the conductor. We apply Ampère's law, integrating around a circle of radius r ,

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

(a) For $r < R$

$$I_{\text{enc}} = J(\pi r^2) = \left(\frac{I}{\pi R^2} \right) (\pi r^2) \rightarrow I_{\text{enc}} = \frac{I r^2}{R^2}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = (2\pi r)B(r) = \mu_0 I_{\text{enc}} \rightarrow (2\pi r)B(r) = \mu_0 \frac{I r^2}{R^2} \rightarrow B(r) = \frac{\mu_0 I r}{2\pi R^2}, \quad r < R.$$

(b) For $R < r$

$$I_{\text{enc}} = I \rightarrow \oint_C \vec{B} \cdot d\vec{\ell} = (2\pi r)B(r) = \mu_0 I \rightarrow B(r) = \frac{\mu_0 I}{2\pi r}, \quad R < r.$$

4. A very long flat conducting sheet of width $2a$ and negligible thickness carries a current I along its length (into the plane of the page). The current is uniformly distributed along the width.

(a) (15 Pts.) Use the Biot-Savart law to find the magnetic field (vector) on the y axis of the figure.

(b) (10 Pts) Suppose the sheet is infinitely wide and each section of width $2a$ carries current I . Use Ampère's law to find the magnetic field on the y axis.

$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln(x + \sqrt{x^2 + y^2}) + C, \quad \int \frac{dx}{x^2 + y^2} = \frac{1}{y} \arctan\left(\frac{x}{y}\right) + C, \quad \int \frac{x dx}{x^2 + y^2} = \frac{1}{2} \ln(x^2 + y^2) + C$$

Solution: (a) Current per unit width is $J = I/2a$. Current through the infinitesimal section dx is $dI = (I/2a)dx$. Magnitude of the Magnetic field created by the infinitesimal current on the y axis is

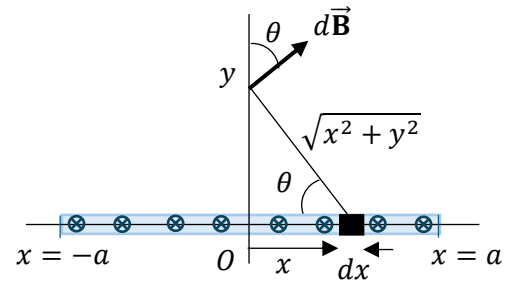
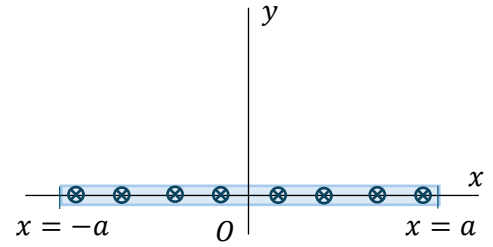
$$|d\vec{B}| = \frac{\mu_0 dI}{2\pi r} = \left(\frac{\mu_0}{2\pi}\right) \left(\frac{I}{2a}\right) \frac{dx}{\sqrt{x^2 + y^2}}$$

$$dB_x = |d\vec{B}| \sin \theta = \frac{\mu_0 I}{4\pi a} \frac{dx}{\sqrt{x^2 + y^2}} \frac{y}{\sqrt{x^2 + y^2}} = \frac{\mu_0 I y}{4\pi a} \frac{dx}{x^2 + y^2}$$

$$B_x = \frac{\mu_0 I y}{4\pi a} \int_{-a}^a \frac{dx}{x^2 + y^2} = \frac{\mu_0 I y}{2\pi a} \int_0^a \frac{dx}{x^2 + y^2} \rightarrow B_x = \frac{\mu_0 I}{2\pi a} \arctan\left(\frac{a}{y}\right)$$

$$dB_y = |d\vec{B}| \cos \theta = \frac{\mu_0 I}{4\pi a} \frac{dx}{\sqrt{x^2 + y^2}} \frac{x}{\sqrt{x^2 + y^2}} = \frac{\mu_0 I}{4\pi a} \frac{x dx}{x^2 + y^2}$$

$$B_y = \frac{\mu_0 I}{4\pi a} \int_{-a}^a \frac{x dx}{x^2 + y^2} = 0, \quad \vec{B} = \frac{\mu_0 I}{2\pi a} \arctan\left(\frac{a}{y}\right) \hat{i}$$



(b) In this case we have an infinitely wide thin sheet of current with uniform current density per unit width $J = I/2a$.

Because of the symmetry of the current distribution, magnetic field is parallel to the x axis as shown in the figure. Applying Ampère's law to the rectangular path shown, we have

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$BL + 0 + BL + 0 = \mu_0 LJ \rightarrow 2BL = \frac{\mu_0 IL}{2a} \rightarrow B = \frac{\mu_0 I}{4a}$$

